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SOME NOTES ON THE TAGUCHI LOSS FUNCTION

By: John McConnell

Dr. Deming was often quoted as saying that it was good management to reduce the variation in any quality characteristic, whether this characteristic be in a state of control or not, and even if few or no defectives are being produced. (1) The six sigma concept introduced in Newsletter 13 demands that the variability of any event in the process ought to be no more than half that allowed by the specifications. Many people found a requirement for such demanding precision difficult to understand, let alone believe. The mind set in most companies is that if the characteristic meets specification or standard, no further improvement is necessary or warranted. Usually, any occurrence outside of a specification or standard is treated as a special cause.

At the time of the industrial revolution the first definition of quality developed was:

> *All characteristics must remain within the go, no-go specifications.*

After Shewhart's contribution, the second definition of quality was:

> *All characteristics must remain stable and within the go, no-go specifications.*

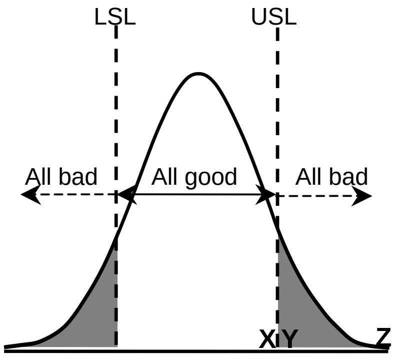
Both Deming's approach and the six sigma methodology are at odds with this definition. Both are much more demanding. Taguchi was able to demonstrate that reducing variation far below specifications was the correct course of action. So was the founder of six sigma, Bill Smith.

In the 1920's Dr. Shewhart noted that quality was essentially an economic problem. Dr. Genichi Taguchi followed this line of thinking. He defined the cost of poor quality as the total loss incurred by society due to variation and poor quality. Taguchi was passionate about quality, to the point where he claimed that the manufacturer of poor quality was worse than a thief. If a thief steals $100 from a neighbour, he has gained and the neighbour has lost, but the net economic impact on society is nil. That $100 will still be invested or spent on goods and services. However, if a manufacturer throws away $100 in rejects and rework, the cost of wasted resources can never be recovered by either the company or by society.

One way to introduce the Taguchi Loss Function is to start with the notion that design engineers, chemists, and biologists do not like variation. They prefer perfect precision and have an optimum value in mind. However, they understand that perfection is impossible and so they create tolerances to delineate the boundaries of permissible variation. Nonetheless, there is always an optimum value (OV). For the sake of simplicity and clarity those characteristics where bigger is always better (horsepower of a racing car) and smaller is always better (weight of a racing car) will be ignored.

Secondly, consider product **X** whose measured characteristic falls just barely inside the upper specification, as noted in Figure 1. Now imagine product **Y**, which is only just outside the upper specification. In terms of their physical properties and their performance in the customer's hands, these two products are likely to be, for all practical purposes, identical. In most cases, the sampling and analytical error involved will be greater than the observed difference between these two outcomes. Yet the methodology used in the most companies will always state that product **Y** is outside specifications and therefore warrants investigation. Now let us add one more product to the chart, product **Z**, which is well outside the specification. The old 'go, no-go' definition of quality states that all products out of specification are equally bad.

Figure 1
The Old View of Variation



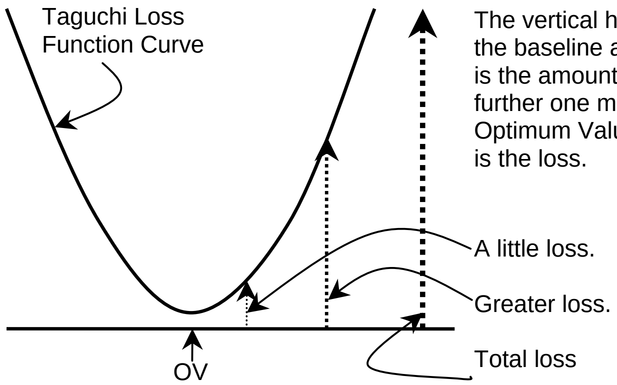
The old view states that **X** is a good result, and that **Y** and **Z** are equally bad.

However, in terms of their physical properties, **X** and **Y** may be nearly identical.

Is there a better way to describe their varying degrees of 'goodness' or 'badness'?

As noted in Figure 2, Taguchi envisioned a loss function curve, usually a parabola, which described the increase in loss on a continuum as outcomes moved away from the optimum value, rather than as a go, no-go situation. The vertical height from the baseline (no loss) to the curve described how the amount of loss increased as outcomes move further away from the optimum value, until complete loss is occasioned. [2]

Figure 2



The vertical height between the baseline and the curve is the amount of loss. The further one moves from the Optimum Value, the greater is the loss.

The Taguchi View of Variation

Taguchi's approach made intuitive sense. Suppose one is using the go, no-go approach and the specifications for pH for a certain process parameter are 6.5 to 7.5. A reading of 7.45 is will be accepted as in specification, but one of 7.55 will be out of specification, even though there is no discernable change in the product. The Taguchi approach sees these readings on a continuum. The target or optimum value is 7.0. Our loss increases continually the further one departs from this optimum. Absolute limits are still necessary for release and compliance reasons, but if loss is seen on a continuum, it opens new perspectives and opportunities to understand and improve the process and product hitherto unnoticed.

The six sigma approach demands abandonment of reliance on the go, no-go specifications. They are still necessary to set absolute limits, but should not be the sole basis for assessing the variation at any stage of the process. In addition, go, no-go specifications should never be used as the basis for process control.

The Taguchi loss function takes several forms. One quadratic form of calculation is shown in equations 1 and 2.

For any given product:

$$Loss_x = k(x-OV)^2 \qquad \text{(Eqn 1)}$$

Where:

x = value of quality characteristic

OV = Optimum Value

k = A proportionality constant (a function of the failure cost structure)

For any given population of products:

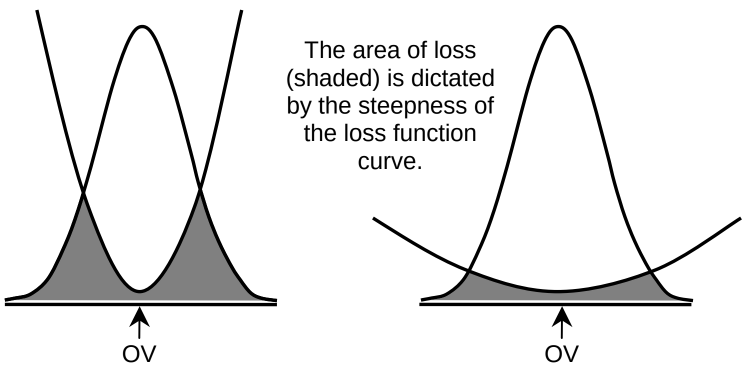
$$Loss_x = k(\sigma_x^2 + (Ave_x - OV)^2) \qquad \text{(Eqn 2)}$$

A simplified version of the *k* factor exists that makes certain assumptions, including that the specifications have been properly set and that any cost of field failure is ignored, is:

$$k = \frac{\text{Cost of failing specification}}{(\text{Average} - \text{specification limit})^2}$$

When the Taguchi Loss Function Curve is overlaid on a distribution, the area of intersect represents the amount of loss, which can be calculated from equation 2 above. Some loss function curves are low and flat, allowing considerable movement away from the optimum value before significant loss is incurred. Others are tall and steep, where even a small deviation from the optimum value can incur considerable loss, as noted in Figure 3. The steepness of this curve is driven by the size of the k factor.

Figure 3

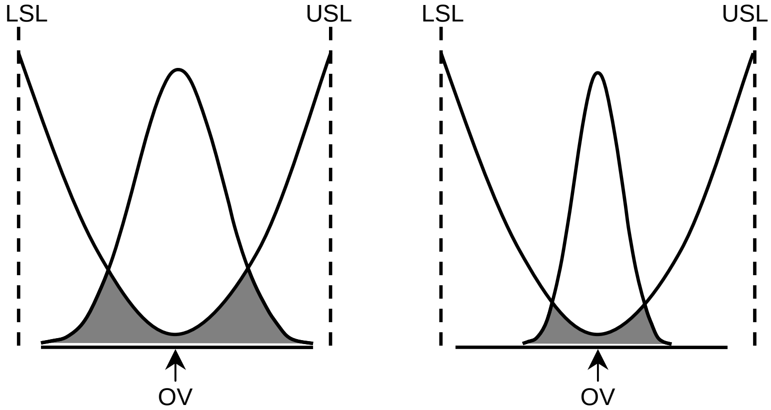


Finding the Area of Loss

In most cases, working on the earlier steps in the process, with particular regard to inputs and set-up of the process, provides significant leverage because the loss function curves are nearly always steeper there. The importance of working upstream in the process can not be overstated. Too often a plant's most talented people can be found working in the middle or latter stages of the process, after variation in the earlier events has already destabilised the process and shrouded cause and effect relationships. Other statistical methods such as DOE will help to isolate those variables that have the greatest impact on quality and costs. A cornucopia of advanced statistical techniques exists, but it beyond the scope of this newsletter to cover them.

Taguchi's approach demonstrates two important ideas. Firstly, it is in concert with Dr. Deming's statement that some loss will be incurred even if all data met specifications and that businesses should always be attempting to reduce variation. Secondly, it demonstrates that as variation is reduced, so too is the amount of loss incurred. It only remains to isolate those variables that provide greatest leverage. These concepts are illustrated at Figure 4.

Figure 4
Reducing Variation Reduces the Area of Loss



Taguchi's concepts led to the development of the third definition of quality:

> *All characteristics should have minimum, stable variation around an optimum value.*

This is in concert with the six sigma concept outlined in Newsletter No. 13. Six sigma states that the nearest specification should be at least six sigma from the process mean. Dr. Taguchi's concept is another way to illustrate how variation impacts on processes and costs. What is clear is that several different approaches to the subject arrive at the same conclusions, as follows. Firstly, variation costs money, even when everything appears to meet specifications. Secondly, reducing variation always reduces costs. Thirdly, not all variables have an equal impact on costs. Some are very significant; others are not. In no small way, a critical part of the Taguchi methods is to discover which variables harbour the greatest potential to reduce costs.

Newsletter No.1 introduces Little's Law, which explains how reduced variation in the flow of material through a process also increases throughput volume, especially when the earliest events in the process are addressed. Variability, it transpires, impacts on both quality and quantity.

Although the loss function curve can be calculated for any variable, it is not necessary to do this in order to grasp the core concepts and to make a start. If it is understood that the early steps in the process generally have steeper loss function curves for both quality and quantity, work to reduce variation can commence there immediately. The flow-on benefits throughout the process can be dramatic.

These understandings are critical in any industry. Variation consumes resources, money and technical people's time. Where a significant proportion of technical people's time is expended investigating out of specification results, time is wasted that could have been spent improving the process and the product. Reducing variation by achieving a stable state and then minimising variability is particularly crucial in highly regulated industries such as the pharmaceutical and aeronautical industries. Close enough is not good enough; nor is being satisfied with meeting specifications.

References:

1. *W.E. Deming,* Out of the Crisis, 1988, MIT
2. *W. Scherkenbach,* The Deming Route to Quality and Productivity, 1988, CEEPress